

# A New Model for Entrainment from Fluidized Beds

A model describing the entrainment of solid particles from gas-fluidized beds of mixed-size particles is proposed. It was tested with the limited experimental data published to date and was shown to be superior to existing models and correlations. The model demonstrates that the concentration of very fine particles in the bed can have a significant effect on entrainment.

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## SCOPE

In columns containing a bed of fluidized solids, the gas leaving the top of the bed carries with it entrained solid particles. It has been shown that as the gas travels upward from the bed surface, the flux of entrained solids decreases until a certain height above the bed surface, called the transport disengaging height (TDH), is reached (Zenz, 1983; Large et al., 1977). The flux of entrained solids remains essentially constant at any height above the TDH. Industrial columns are thus designed so that the gas exits the column above the TDH.

Pollution control regulations as well as the maintenance of fluidization quality and high solids costs in some cases (e.g., catalytic processes) make it necessary to recover the solids entrained out of the column. Proper design of solids recovery equipment such as cyclone trains thus requires accurate predictions of both the flux and the size distribution of the solids entrained above the TDH.

Numerous correlations have been proposed for the calcula-

tion of the flux of entrained solids (Wen and Chen, 1982). However, these correlations cannot be used outside of the range of conditions over which they were developed (Zenz, 1983; Hazlett, 1985). Moreover, to our knowledge, none of these correlations can predict the size distribution of entrained material. Models have been proposed by Zenz and Weil (1958), George and Grace (1978), and Gugnoni and Zenz (1980) to calculate both the flux and the size distribution of the solids entrained above the TDH. Although some of these models are quite useful, none can describe accurately all available experimental data.

The objective of the work presented here was to develop and test a model that would allow designers to predict accurately the flux and the size distribution of the solids entrained above the TDH. Such a model should be a general model based on simple physical principles; it could thus be applied over a wide range of experimental conditions.

## CONCLUSIONS AND SIGNIFICANCE

A simple physical model was developed to predict both the flux and the size distribution of the solids entrained above the transport disengaging height of fluidized beds. This model assumes that for each particle size, the flux of entrained solids is limited by both the choking load and the flux of solids ejected from the bed surface. It also assumes that the gas will preferentially entrain the smallest particles, thus maximizing the total entrainment flux.

This model was successfully tested with the rather limited amount of published entrainment data. The best choking load

correlation was found to be the new Zenz (1983) correlation.

The model presented in this paper shows that the flux of entrained solids depends greatly on the concentration of the smallest particles in the bed. Although these particles might constitute less than 1 wt. % of the bed particles, they constitute most of the particles entrained out of the column. To obtain accurate predictions of the entrainment flux, it is essential to evaluate how solids losses past the particle recovery equipment and attrition will affect the concentration of these fines in the bed.

In most cases, the flux of solids entrained above the TDH is to a large degree dependent on the flux of solids ejected from the bed surface. From a practical point of view, this means that any reduction of the flux of solids (and specially fines) ejected from the bed surface might result in a significant reduction of the flux of solids entrained above the TDH. The model can

be used to predict the reductions in entrainment fluxes that can be achieved by installing deentrainment devices just above the bed surface, moving a cyclone return leg to return the solids deep into the bed instead of above it, or using any other means of reducing the flux of particles ejected into the free-board.

## REVIEW OF PUBLISHED MODELS

### Zenz-Weil Model

The basic assumption of this model is that the flux of solids entrained above the transport disengaging height (TDH) is equal to the maximum solids flux which could be carried in a pneumatic transport line operating at the same superficial gas velocity as the fluidized bed column (Zenz and Weil, 1958). This maximum solids flux is also called the choking load.

Thus, for each particle size cut  $[d_{pi}, d_{pi} + \Delta d_{pi}]$ , the choking load for monosize particles  $G_i$  is calculated from correlations developed for pneumatic transport and the flux of particles of that size entrained above the TDH is given by:

$$F_{oi} = x_{Bi} G_i \quad (1)$$

where  $x_{Bi}$  is the wt. % of the particle size cut  $[d_{pi}, d_{pi} + \Delta d_{pi}]$  in the bed solids.

The total flux of solids entrained above the TDH is then given by:

$$F_{\infty} = \sum F_{oi} \quad (2)$$

The size distribution of the entrained material is given by:

$$x_i = F_{oi}/F_{\infty} \quad (3)$$

(Note that the particles with terminal velocities larger than the superficial gas velocity will not be entrained, as their choking loads are equal to zero.)

The Zenz-Weil model was applied to experimental data collected by Large et al. (1976), Zenz and Weil (1958), Bachovchin et al. (1979), Fournol et al. (1973), and Tweddle et al. (1970). These experimental data include average particle diameters ranging from 58 to 700  $\mu\text{m}$  and particle densities ranging from 830 to 2,630  $\text{kg}/\text{m}^3$ . These particles thus belong to groups A and B of Geldart's classification. No data were available for cohesive solids (group C) and large particles (group D). A correlation given by Zenz and Weil was used to calculate choking loads. The calculated and experimental values of the flux of solids entrained above the TDH are compared in Figure 1. It can be seen that in most cases the calculated flux was within an order of magnitude of the actual experimental value.

Large et al. (1976) used three sands which had mean particle diameters of about 140  $\mu\text{m}$  and differed only by their under-

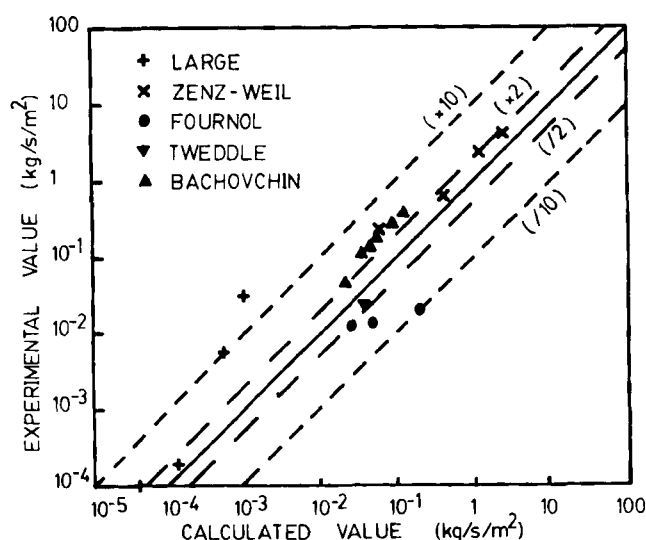


Figure 1. Flux of solids entrained above the TDH. Comparison of experimental data with values calculated by Zenz-Weil (1958) model.

TABLE 1. COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES OF THE FLUX OF SOLIDS ENTRAINMENT ABOVE THE TDH (DATA FROM LARGE ET AL., 1976)

Solids (Sand)	$V_g$ m/s	Experimental $\text{kg/s} \cdot \text{m}^2$	Value of $F_{\infty}$	
			Zenz-Weil (1958) Model $\text{kg/s} \cdot \text{m}^2$	Gugnoni-Zenz (1980) Model $\text{kg/s} \cdot \text{m}^2$
S <sub>1</sub>	0.3	$1.8 \times 10^{-4}$	$1.4 \times 10^{-4}$	$7.2 \times 10^{-3}$
S <sub>2</sub>	0.3	$3.0 \times 10^{-2}$	$9.8 \times 10^{-4}$	$6.2 \times 10^{-2}$
S <sub>3</sub>	0.3	$5.5 \times 10^{-3}$	$5.8 \times 10^{-4}$	$3.3 \times 10^{-2}$
S <sub>3</sub>	0.2	$10^{-4}$	$4.3 \times 10^{-5}$	$5.3 \times 10^{-3}$

37  $\mu\text{m}$  fines content wt. %: 0.00 for sand S1, 1.12 for sand S2, 0.62 for sand S3. Table 1 shows that the entrainment fluxes obtained under identical conditions with the three sands varied by over two orders of magnitude. These results demonstrated the important effect of fines contents on entrainment. By contrast, entrainment fluxes predicted with the Zenz-Weil model varied by less than one order of magnitude. Table 2 gives experimental and calculated values of the mean particle diameter of the solids entrained above the TDH for Large's data. It can be seen that the Zenz-Weil model gave rather poor predictions of the size distribution of the entrained solids. In particular, according to Large's experimental data for the sand S3, the mean particle diameter of the entrained solids actually increased when the gas velocity was reduced from 0.3 to 0.2 m/s, while the Zenz-Weil model predicted a decrease in mean particle diameter.

### George-Grace Model

George and Grace (1978) assumed that for each particle size with a terminal velocity smaller than the superficial gas velocity, the flux entrained above the TDH was equal to the flux ejected from the bed surface. Although this model tends to overestimate the entrained flux by several orders of magnitude in most cases, to our knowledge it was the first model to take into account the effect of the flux of solids ejected from the bed surface on the flux of solids entrained above the TDH.

### Gugnoni-Zenz Model

This model (Gugnoni and Zenz, 1980) proceeds in two steps. The first step is the calculation of the size distribution of the solids entrained above the TDH. The authors used a procedure identical to the Zenz-Weil model to compute the concentration of each particle size in the solids entrained above the TDH from its choking load and its concentration in the bed solids:

$$x_i = x_{Bi} G_i / \left( \sum x_{Bi} G_i \right) \quad (4)$$

TABLE 2. COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES OF THE LOG MEAN PARTICLE DIAMETER OF THE SOLIDS ENTRAINMENT ABOVE THE TDH (DATA FROM LARGE ET AL., 1976)

Solids (Sand)	$V_g$ m/s	Experimental $\mu\text{m}$	Value of $\bar{d}_{pm}$	
			Zenz-Weil (1958) Model $\mu\text{m}$	Gugnoni-Zenz (1980) Model $\mu\text{m}$
S <sub>1</sub>	0.3	50	58	51
S <sub>2</sub>	0.3	21	33	19
S <sub>3</sub>	0.3	26	43	26
S <sub>3</sub>	0.2	34	36	21

They then computed the mean diameter of the particles entrained above the TDH.

In the second step, the total flux of solids entrained above the TDH is calculated with a formula based on the mean diameter of the particles entrained above the TDH and the difference between the superficial gas velocity and the minimum fluidization velocity of the bed solids. It should be noted that most correlations for the calculation of the flux of solids ejected from the bed surface also use the difference between the superficial gas velocity and the minimum fluidization velocity (Wen and Chen, 1982; Pemberton, 1982).

Although it is not claimed by Gugnoni and Zenz—who just give empirical correlations based on analogies with distillation—an interpretation of their procedure is that the flux of solids entrained above the TDH is assumed to be limited by choking in order to compute the size distribution of those solids. The flux of solids entrained above the TDH is then calculated by assuming that it depends on the flux of solids ejected from the bed surface.

The Gugnoni-Zenz model was applied to experimental data collected by Large et al. (1976), Zenz and Weil (1958), Bachovchin et al. (1979), Fournol et al. (1973), and Tweddle et al. (1970). The correlation used to compute choking loads was the Gugnoni-Zenz correlation as modified by Zenz (1983) to account for the effect of particle density. The calculated and experimental values of the flux of solids entrained above the TDH are compared in Figure 2. It can be seen that in most cases the calculated flux was within an order of magnitude of the actual experimental value. As the calculated flux was always larger than the experimental flux, the Gugnoni-Zenz model would lead to a conservative design of the solids recovery equipment.

In the case of Large's data, the results obtained with the Gugnoni-Zenz model were similar to the results obtained with the Zenz-Weil model. Entrainment fluxes as predicted by the model for the three sands at a superficial velocity of 0.3 m/s varied by less than one order of magnitude, while the experimental fluxes varied by over two orders of magnitude (Table 1). Although the Gugnoni-Zenz model gave better predictions of the mean diameter of solids entrained above the TDH than the Zenz-Weil model, it also failed to account for the increase of this mean diameter when the gas velocity was reduced from 0.3 to 0.2 m/s (Table 2, sand S3).

## PROPOSED MODEL

### Basic Assumptions

A model is proposed to describe entrainment above the TDH. It is based on three basic assumptions:

1. For each particle size, the flux of solids entrained above the TDH is limited by choking; i.e., it cannot be larger than the fraction of the total choking load which is attributed to that particle size when the column above the TDH is assumed to behave as a pneumatic transport line.

2. For each particle size, the flux of solids entrained above the TDH cannot be larger than the flux of solids ejected from the bed surface.

Assumptions 1 and 2 would be sufficient to calculate the flux of entrained solids for monosize particles. In the case of multi-size particles, however, an extra assumption is required to define both the size distribution and the flux of the entrained material:

3. It is assumed that the size distribution of the entrained material will be such that it will maximize the entrained flux, namely, the gas will preferentially entrain the smallest particles.

### Calculation Procedure

Calculation of the entrained flux requires the following data:

- The accurate particle size distribution of bed material and the terminal velocity for each particle size cut.
- The superficial gas velocity in the column.
- The total flux of solids ejected from the bed surface (extrapolated from experimental data or calculated with a correlation).

It also requires a choking load model or correlation.

The iterative calculation proceeds according to the following steps:

**Step 0. Calculation of Initial Values.** The flux ejected from the bed surface is calculated for each particle size cut from its concentration in the bed solids and the total flux of solids ejected from the bed surface:

$$F_{oi} = x_{Bi} F_o \quad (5)$$

(Note: If there is segregation by particle size in the bed,  $x_{Bi}$  should be replaced by the weight fraction of particle size cut  $i$  in the bed surface layers.)

A first approximation of the size distribution of the material entrained above the TDH is given by:

$$x_i = x_{Bi} \quad \text{if } U_n < V_g \quad (6)$$

$$x_i = 0 \quad \text{if } U_n > V_g \quad (7)$$

**Step 1. Calculation of the Choking Load.** The total choking load  $W$ , which depends on the size distribution of the entrained solids, is calculated from a correlation or a model. The fraction of the choking load that is attributed to each particle size is then given by:

$$W_i = x_i W \quad (8)$$

**Step 2. Entrained Flux for Each Particle Size.** According to assumptions 1 and 2, the entrained flux will be given by:

$$F_{oi} = W_i \quad \text{if } W_i < F_{oi} \quad (9)$$

$$F_{oi} = F_{oi} \quad \text{if } W_i > F_{oi} \quad (10)$$

**Step 3. Size Distribution and Total Flux of Entrained Solids.** The total flux of entrained solid is given by:

$$F_o = \sum F_{oi} \quad (11)$$

The size distribution of the entrained solids is then given by:

$$x_i = F_{oi} / F_o \quad (12)$$

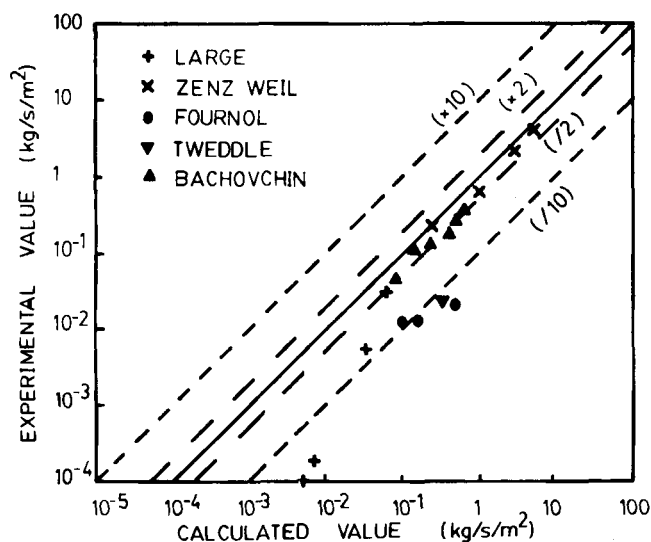


Figure 2. Flux of solids entrained above the TDH. Comparison of experimental data with values calculated by Gugnoni-Zenz (1980) model.

**Step 4. Adjustment of the Size Distribution.** According to assumption 3, the fines content of the entrained solids is maximized:

$$x_i = x_i(V_g - U_n)^n \quad (13)$$

and the concentrations are normalized:

$$x_i = x_i / \left( \sum x_i \right) \quad (14)$$

The whole calculation is then repeated from step 1 to step 4. Fifty iterations were used for all the entrainment flux calculations of this study, although convergence was usually reached in about ten iterations. The exponent  $n$  was reduced gradually from 5.0 for the first iteration to 0.01 for the fiftieth iteration.

Thus,  $(V_g - U_n)^n$  is just used to increase the weight fraction of the smaller particles in the solids entrained above the TDH. It is a convergence factor that for the first few iterations (for high values of  $n$ ) eliminates practically all the larger particles by making their weight fraction  $x_i$  very small. For the last iterations it becomes practically equal to 1 and thus does not affect the size distribution.

To summarize the calculation procedure: The gas is assumed to entrain as many of the smallest particles as it can; it cannot entrain more particles of a given size than are ejected from the bed surface and it cannot entrain more than the choking load corresponding to that particle size. The same procedure is repeated for increasing particle sizes. An iterative procedure is required as, for mixed size particles, the choking load of one particle size is affected by the actual size distribution of the entrained solids (i.e., it is affected by the entrained flux of the other particle sizes).

#### Choking Load Correlations Used in This Study

As shown above, the entrainment model must use a choking load correlation. Two such correlations were used in this study.

**Zenz Correlation.** Zenz (1983) gives a plot that can be expressed as:

$$x = kY^p \quad (15)$$

with

$$x = \frac{G_i}{V_g \rho_g} \text{ and } Y = \frac{(V_g - U_n)}{\rho_p (gd_{pi})^{0.5}} \quad (16)$$

and (SI units):

$$k = 3.06 \cdot 10^6, p = 4.40 \text{ for } Y < 0.01 \quad (17)$$

$$k = 5.43 \cdot 10^5, p = 3.05 \text{ for } Y > 0.01 \quad (18)$$

with:  $W_i = x_i G_i$

**Yang-Leung Correlation.** Yang (1975, 1983) proposed a choking load correlation for monosize particles that was adapted by Leung and Wiles (1976) to the case of mixed size particles. This correlation is defined by the following equations:

$$f_{pc} = 6.81 \cdot 10^5 (\rho_g / \rho_p)^{2.2} \quad (20)$$

$$\left[ \sum x_n (V_g - U_n) \right]^2 = 2gD(\epsilon_c^{-4.7} - 1) / f_{pc} \quad (21)$$

$$W_i = x_i W = \rho_p x_n (1 - \epsilon_c) (V_g - U_n) \quad (22)$$

$$\sum x_n = \sum x_i = 1 \quad (23)$$

We found that this correlation could be simplified by eliminating  $x_n$ , which is the actual weight fraction in the riser of the particle cut and which can be expressed as:

$$Z = \sum [x_i / (V_g - U_n)] \quad (24)$$

$$x_n = x_i / [Z(V_g - U_n)] \quad (25)$$

This gives:

$$(\epsilon_c^{-4.7} - 1) = f_{pc} / (2gDZ^2) \quad (26)$$

$$W = \rho_p (1 - \epsilon_c) / Z \quad (27)$$

$$W_i = x_i W \quad (28)$$

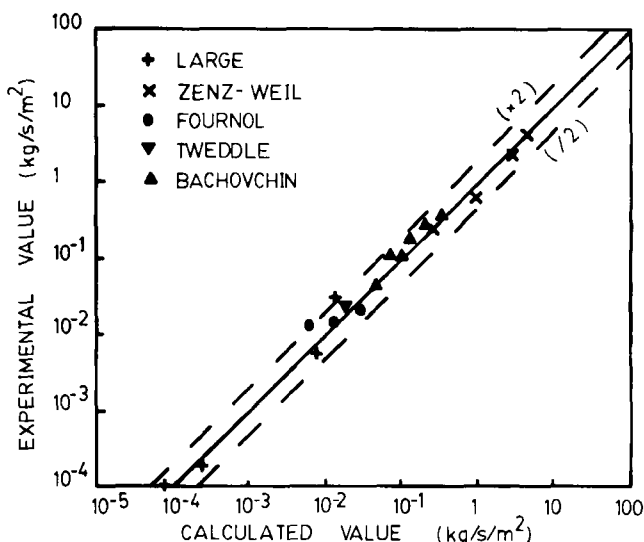
#### TEST OF THE PROPOSED MODEL WITH EXPERIMENTAL DATA FROM LITERATURE

##### Test Using Experimental Values of the Flux of Solids Ejected from the Bed Surface

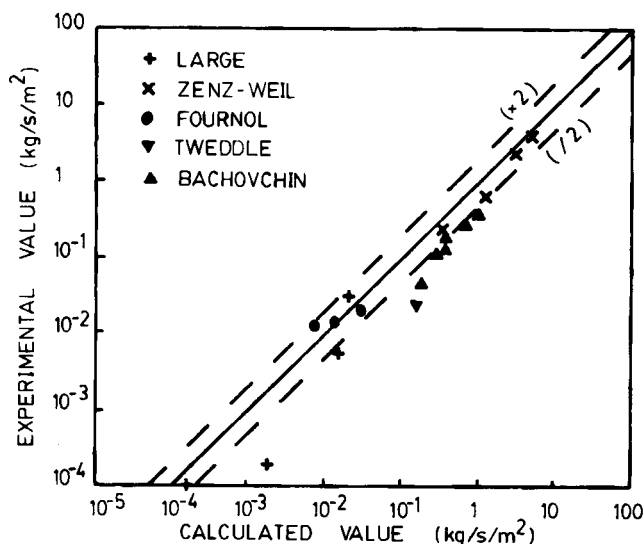
The model was tested with data for which experimental values of the flux of solids ejected from the bed surface were available. Experimental values of the flux of solids ejected from the bed surface are usually obtained by extrapolation and thus depend on the extrapolation procedure. For the experimental data of Bachovchin (1979), Tweddle (1970), and Zenz and Weil (1958), an average of the extrapolated values given by Chen (1981) and Pemberton (1982) was used. For the data of Fournol et al. (1973), Pemberton's estimates were the only ones available. Large et al. (1976) provided accurate extrapolated values of the flux of solids ejected from the bed surface for his own data.

The flux of solids entrained above the TDH was then calculated with the proposed model. Figure 3 compares the experimental values to the values calculated with the model and Zenz (1983) correlation for the choking load. The calculated values of Figure 4 were obtained with the Yang-Leung correlation for the choking load (Yang, 1983; Leung and Wiles, 1976). A comparison of Figures 3 and 4 shows that the Zenz correlation for the choking load gave much better results than the Yang-Leung correlation.

A comparison of the results obtained with the proposed model (Figure 3) and the results obtained with the Zenz-Weil model (Figure 1) and the Gugnoni-Zenz model (Figure 2) clearly demonstrates that the proposed model is quite superior to these earlier models. While the Zenz-Weil and Gugnoni-Zenz models gave predicted values of the flux of entrained solids that could be off by over an order of magnitude, all the



**Figure 3. Flux of solids entrained above the TDH. Comparison of experimental data with values calculated with the proposed model, experimental values of the flux ejected from the bed surface, and Zenz correlation for choking.**



**Figure 4. Flux of solids entrained above the TDH. Comparison of experimental data with values calculated with the proposed model, experimental values of the flux ejected from the bed surface, and Yang-Leung correlation for choking.**

values predicted with the proposed model did not differ from the experimental values by more than a factor of 2, i.e., the actual experimental value was always bigger than half the calculated value and smaller than double the calculated value.

#### Test with Large's Data: Prediction of the Mean Diameter of Entrained Particles

As shown earlier, both the Zenz-Weil and the Gugnoni-Zenz models were completely unable to interpret the experimental results obtained by Large et al. (1976) (see Tables 1 and 2).

Large et al. used three sands that had mean particle diameters of about 140  $\mu\text{m}$  and that differed only by their fines content. They found that the entrainment fluxes obtained under identical conditions with the three sands varied by over two orders of magnitude. As shown in Table 3, the values predicted by the proposed model were in good agreement with Large's experimental data.

To our knowledge, Large et al. are the only researchers who reported both accurate values of the flux ejected from the bed surface and accurate size distributions of the material entrained above the TDH. Table 4 demonstrates that the proposed model gave accurate predictions of the mean diameter of the particles entrained above the TDH. In particular, the proposed model accurately predicted that when the superficial gas velocity was

**TABLE 3. COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES OF THE FLUX OF SOLIDS ENTRAINMENT ABOVE THE TDH (DATA FROM LARGE ET AL., 1976)**

Solids (Sand)	$V_g$ m/s	Value of $F_{\infty}$	
		Experimental kg/s $\cdot$ m <sup>2</sup>	Proposed Model* kg/s $\cdot$ m <sup>2</sup>
S <sub>1</sub>	0.3	$1.8 \times 10^{-4}$	$2.5 \times 10^{-4}$
S <sub>2</sub>	0.3	$3.0 \times 10^{-2}$	$1.4 \times 10^{-2}$
S <sub>3</sub>	0.3	$5.5 \times 10^{-3}$	$7.8 \times 10^{-3}$
S <sub>3</sub>	0.2	$10^{-4}$	$0.65 \times 10^{-4}$

\*Using Zenz choking load correlation.

**TABLE 4. COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES OF THE LOG MEAN DIAMETER OF THE SOLIDS ENTRAINMENT ABOVE THE TDH (DATA FROM LARGE ET AL., 1976)**

Solids (Sand)	$V_g$ m/s	Value of $\bar{d}_{pm}$	
		Experimental $\mu\text{m}$	Proposed Model* $\mu\text{m}$
S <sub>1</sub>	0.3	50	48
S <sub>2</sub>	0.3	21	22
S <sub>3</sub>	0.3	26	28
S <sub>3</sub>	0.2	34	33

\*Using Zenz choking load correlation.

reduced from 0.3 to 0.2 m/s, the mean diameter of the entrained particles actually increased. Although these experimental results might at first seem surprising (and other models were unable to predict them), they can easily be explained with the proposed model. As the gas velocity was reduced to 0.2 m/s, the reduction of the flux of particles ejected from the bed surface was much greater than the reduction of the solids carrying capacity of the gas in the dilute phase above the bed. Consequently, the gas could not only carry all the fines ejected from the bed surface, but it had enough solids carrying capacity left to carry some of the coarser solids above the TDH. At the higher superficial gas velocity (0.3 m/s), the amount of solids ejected from the bed surface was so large that the gas could not even carry above the TDH all the fines ejected from the bed.

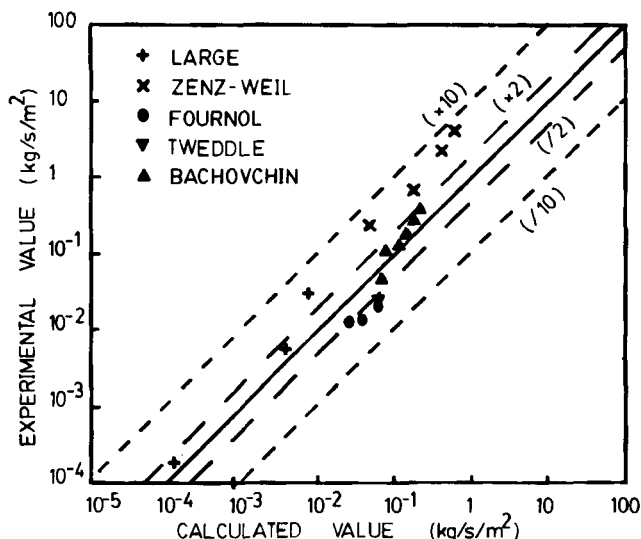
#### Test Using Predicted Values of the Flux of Solids Ejected from the Bed Surface

In most practical applications, no experimental value is available for the flux of particles ejected from the bed surface. It is therefore necessary to use some correlation to estimate the flux of particles ejected from the bed surface. Three of the correlations available in the literature—the Wen-Chen (1982) correlation, and Pemberton's (1982) bubble wake and nose models—were tested with the available experimental data (Bachovchin et al., 1979; Fournol et al., 1973; Gugnoni and Zenz, 1980; Large et al., 1976; Tweddle et al., 1970; Zenz and Weil, 1958). The best predictions were obtained with Pemberton's nose model, which estimates the flux of particles ejected from the bed surface with the following formula:

$$F_o = 3d_p(1 - \epsilon_{mf})(V_g - U_{mf})/d_B \quad (29)$$

The minimum fluidization velocity was estimated with the Wen-Yu correlation (Wen and Yu, 1966) and the bubble diameter was estimated with Darton's correlation (Darton, 1979).

Figure 5 compares experimental values of the flux of solids entrained above the TDH with values obtained with the proposed model when Zenz's correlation for the choking load and Pemberton's bubble nose model for the flux of solids ejected from the bed surface were used. A comparison of Figure 5 with Figure 3, for which experimental values of the ejected flux were used to calculate the entrained flux, indicates that there was a significant loss in the accuracy of the model predictions when estimated values of the ejected flux were used. However, a comparison of Figure 5 with Figures 1 and 2 demonstrates that even when a correlation was used to evaluate the ejected flux, the proposed model gave more accurate predictions of the flux of solids entrained above the TDH than the correlations available in the literature. When Pemberton's bubble nose model was used to predict the flux of solids ejected from the bed surface, the values of the flux of solids entrained above the TDH were well within an order of magnitude of the experimental values (Figure 5) and neither the Zenz-Weil correlation



**Figure 5.** Flux of solids entrained above the TDH. Comparison of experimental values with values calculated with the proposed model, values for the flux ejected from the bed surface estimated from Pemberton's bubble nose model, and Zenz correlation for choking.

(Figure 1) nor the Gugnoni-Zenz correlation (Figure 2) could match this accuracy.

#### NOTATION

- $d_b$  = bubble diameter, m  
 $d_{pi}$  = particle diameter for size cut  $i$ , m  
 $d_{pm}$  = average particle diameter of solids entrained above the TDH, m  
 $f_{pc}$  = friction factor at choking  
 $F_o$  = flux of particles ejected from bed surface,  $\text{kg/s} \cdot \text{m}^2$   
 $F_{oi}$  = flux of particles of size  $i$  ejected from bed surface,  $\text{kg/s} \cdot \text{m}^2$   
 $F_\infty$  = flux of solids entrained above the TDH,  $\text{kg/s} \cdot \text{m}^2$   
 $F_{ei}$  = flux of solids of size  $i$  entrained above the TDH  
 $g$  = gravity constant,  $\text{m/s}^2$   
 $G_i$  = choking load of particle of size  $i$  when all particles have the same size,  $\text{kg/s} \cdot \text{m}^2$   
 $k$  = constant, Eq. 15  
 $n$  = variable exponent, Eq. 13  
 $p$  = exponent, Eq. 15  
**TDH** = transport disengaging height, m  
 $U_{mf}$  = minimum fluidization velocity, m/s  
 $U_{ti}$  = terminal velocity of particles of size  $i$ , m/s  
 $V_g$  = superficial gas velocity, m/s  
 $\bar{W}$  = total choking load for mixed-sized solids,  $\text{kg/s} \cdot \text{m}^2$   
 $W_i$  = choking load of particles of size  $i$  when other sizes are present,  $\text{kg/s} \cdot \text{m}^2$

- $x_{Bi}$  = weight fraction of solids of size  $i$  in the bed  
 $x_i$  = weight fraction of solids of size  $i$  in the entrained flux above the TDH  
 $x_H$  = actual weight fraction of solids of size  $i$  in the riser  
 $X$  = variable, Eq. 16  
 $Y$  = variable, Eq. 16  
 $Z$  = variable, Eq. 24

#### Greek Letters

- $\epsilon_c$  = voidage at choking  
 $\epsilon_{mf}$  = voidage at minimum fluidization conditions  
 $\rho_g$  = gas density,  $\text{kg/m}^3$   
 $\rho_p$  = particle density,  $\text{kg/m}^3$

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